Comparison of Hydrocarbon Volumetric Calculation between Cell-Based Model and Numerical Integration

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Abstract

Volume estimation for hydrocarbon reserves is a challenging yet pivotal task in engineering for exploration and production. Advances in technology now enable us to compute volume integration using programming computation. Various approaches using numerical integration, including the trapezoidal, pyramidal, and Simpson's rule, along with cell-based models as comparative methods can be used for the calculation of hydrocarbon volume. In this study, original oil in place (OOIP) is employed to determine reserve oil volume. The OOIP values obtained are 8.55 million m$^3$ through cell-based calculations, 8.63 million m$^3$ via the trapezoidal approach, 8.58 million m$^3$ using the pyramidal method, and 8.57 million m$^3$ with Simpson's 3/8 rule. The relative error ratio percentages between the cell-based model as the reference value and the numerical integration calculations as the measured values are 0.93% for the trapezoidal method, 0.35% for the pyramidal method, and 0.23% for Simpson's 3/8 rule. Simpson's 3/8 rule demonstrates the closest mathematical result to the cell-based model. Within this margin of error, the methodologies have been demonstrated to proficiently compute hydrocarbon reserves from real data through simplified and abbreviated processes.

Keyword: cell-based model, hydrocarbon volume, numerical integration

1. Introduction

Technology advancement nowadays could shorten the duration, simplify a complicated calculation, and reduce uncertainty using computational sciences [1]. Computational Science and Engineering revolutionizes research, industry, and education through algorithmic innovations and software systems, facilitating discoveries and advancements across various disciplines and scales [2]. This computational science can be implemented in the calculation of the volume of oil and gas reserves. The volume calculation approach will be carried out using three methods, namely the trapezoidal method, the pyramid method, and Simpson's 3/8 rule [3].

The conventional volumetric calculation method is indeed widely used but has limitations, particularly in accounting for correlations among reservoir variables. Ignoring these correlations can lead to inaccurate estimations of hydrocarbon volumes [4]. Inaccuracies in volume prediction may result in significant losses, encompassing time inefficiencies, technological resource wastage, and economic ramifications [5]. Furthermore, advanced modeling software typically breaks down the entire reservoir into millions of smaller cubes to estimate hydrocarbon reserve volumes. There are numerous steps involved in volume calculation using modeling software, making it somewhat complex [6].

Another challenge in calculating hydrocarbon reserves is the subsurface geometry of the oil and gas compartments. The geometry of hydrocarbon reserves constitutes complex, unstructured measured data [7], especially in the geological setting of Indonesia, where the complexity is more advanced compared to other places [8]. The subsurface geological structure exhibits asymmetry, thereby introducing additional challenges in accurately predicting the exact volume. Thus, calculating the volume of hydrocarbon reserves is not a simple task, yet it is very important. One of the fundamental challenges in the field of petroleum exploration and production engineering lies in accurately estimating the volume of reserves [9].

With technological advancements, we can apply volume integration calculations using
computation to achieve more accurate results. Computational calculations can handle unstructured 3D geometries, which are almost impossible for human beings to calculate manually. The previous similar research utilized model-generated subsurface data, which tends to be more symmetrical and regular [10]. Moreover, this research aimed to utilize real unstructured subsurface data obtained from seismic acquisition. To achieve the best results, adjustments and refinements were made to accommodate the challenges inherent in the complexity of the data itself.

In this study, two methodologies are employed: conventional calculation using initial software and numerical integration. The conventional approach entails the utilization of Schlumberger Petrel software's cell-based model, which is deemed a benchmark due to its widespread usage. Conversely, numerical integration involves employing mathematical equations to compute volume, achieved by partitioning the compartment into geometric subsets. Both conventional and numerical integration outputs are presented in terms of hydrocarbon reserve volume, reflecting the comprehensive computation of pertinent hydrocarbon parameters, with particular emphasis on aligning outcomes with those obtained from Petrel software.

The automated calculation utilizing integration approximation is poised to reduce the procedural steps in conventional methods while upholding the requisite level of accuracy. To account for errors inherent in the modeling approach, three different numerical approximations are employed for calculation. By employing three computational integration methods and comparing the results with the benchmark output, it is anticipated that one integration method will emerge as the most accurate.

2. Methods

An overview of the theories underlying the methodologies used to compute hydrocarbon volume. There are cell-based models and numerical techniques like the trapezoidal, pyramidal, and Simpson’s rule.

First, the approach using the initial software, Petrel by Schlumberger, can be explained as a cell-based model. A cell-based model itself consists of a large number of cells or pillars that make up the geological subsurface model [11]. Grids can be either organized or unstructured [12]. As seen in Figure 1, structured grids are widely applied for petroleum geological operations. Using a cell-based model when creating geological models gives you more freedom when it comes to inserting faults or other stratigraphic models. Utilizing Petrel software, all parameters affecting the geological setting have been taken into account, and the system automatically transforms them into the cell-based model. High precision and flexibility are provided by the cell-based model in responding to model user input [13].

Basically, the calculation based on this software follows the system built by the developer. This method is mostly used to display the subsurface model and calculate oil reserves. In this research, considering this fact, the output from the cell-based model is determined as the reference or benchmark for other approaches.

Transitioning to the other proposed method, since the three approaches involve numerical integration, an approximation is necessary for complex scenarios, such as those encountered in the research study [15]. The trapezoidal rule is one of the numerical integration methods used to estimate the integration of an area under a curve. The area must be divided into a certain number of equal-width "partitions" in order to apply the trapezoidal rule approach. When the upper section of the curve forming the trapezoidal was replaced by a chord, the trapezium was formed. Next, calculate an estimate of the area that the trapezium formula shapes on the divided region [16]. The numerical estimate of the area beneath the curve is obtained by adding these partition approximations [17]. The left and right sums are averaged according to this rule.

Trapezoidal method volume \( V_{trap} \) approximation can be calculated as in equation
Number of partitions is needed in this calculation; a bigger number of partitions will lead to a better result. The width \( h_{\text{part}} \) of each section \( (A_x) \) is equal in all of the partitions.

\[
V_{\text{trap}} = \frac{h_{\text{part}}}{2} \sum_{x=1}^{N} A_{x-1} + A_x
\]

(1)

Calculate the Pyramidal method volume \( V_{\text{pyr}} \) is similar with trapezoidal method as in equation (2). The volume will be divided into some partitions. Dividing the 3D plane into several parts with equal distances, with each section resembling a trapezoid [18].

\[
V_{\text{pyr}} = \frac{h_{\text{part}}}{3} \sum_{x=1}^{N} A_{x-1} + A_x + \frac{2}{3} A_{x-1} \times A_x
\]

(2)

The last one, quadratic polynomials are employed in Simpson’s Rule to approximate a function’s integral (i.e., a parabolic curve than a linear function). Given that it is typically more precise than the other numerical methods, Simpson’s Rule tends to be quite effective [10, 19]. The formula for Simpson’s rule can be found by integrating a third-order Lagrange interpolating polynomial suited to the function at three evenly spaced areas [20]. Simpson’s Rule volume \( V_{\text{simp}3/8} \) approximation can be calculated as in equation (3).

\[
V_{\text{simp}3/8} = \frac{3h_{\text{part}}}{8} \left[ A_0 + 3 \sum_{x=1}^{N-1} A_x + 2 \sum_{x=3,6,9,...}^{N-3} A_x + A_N \right]
\]

(3)

The number of iterations in this study is not predetermined but repeated until consecutive iteration results converge within a small margin. However, there is a condition that must be fulfilled which is each method of numerical integration requires an odd number of iterations due to compatibility with the integration method.

The majority of geological structures are asymmetric or faulted, requiring the simultaneous calculation of volumes with numerical integration. In this situation, it’s critical to remember that the computed difference between those approaches serves as a test of the method’s applicability, one of which could be derived from equation (4) [21].

When the precondition is met, Simpson’s rule is more effective at identifying asymmetric subsurface structures [22 - 24].

\[
|V_{\text{trap}} - V_{\text{simp}3/8}| \leq 0.2V_{\text{simp}3/8}
\]

(4)

A comparison between the trapezoidal and Simpson methods has been stated, but finding a similar condition for the pyramidal approximation is difficult due to limited literature on pyramidal numerical integration. However, as stated by Chen et al., the pyramidal method has its own virtues. This high convergence rate is due to the special structure of the partition, which leads to the cancellation of some errors [25].

After defining the subsurface model and calculating the reservoir geometry using the cell-based model and numerical integration, the next step is to determine the hydrocarbon reserves. In this study, the volumetric approach was selected to estimate the Original Oil in Place (OOIP), which represents the amount of hydrocarbon initially stored in the reservoir before production. Since this technique does not take into consideration its complications with reservoir heterogeneity, reserve estimates are frequently high [26].

OOIP based on volumetric method can be determined as equation (5) where the total oil volume \( V_o \) was affected by some factors such as scope of oil distribution on the field refer as area \( A \), net-pay thickness means oil fills the reservoir to a certain thickness \( h \), ability of rocks to move water often known as the formation’s porosity \( \phi_f \), ratio of the fluid’s volume to the rock’s pore space named water saturation \( S_w \), and formation volume factor \( B_o \).

\[
V_o = \frac{A \cdot h \cdot 7758 \cdot \phi_f \cdot (1 - S_w)}{B_o}
\]

(5)

The output from the cell-based model for hydrocarbon reserve calculation is already in the form of OOIP. OOIP is automatically calculated by the system using equation (5) after retrieving the area \( A \) and thickness \( h \) of the oil reservoir. Similarly, the OOIP output from the other numerical method is also derived from the calculation of equation (5), with slight differences in the volume components. In this case, it is not derived from the area and thickness anymore, but rather from the volume resulting from the numerical integration calculation of each approximation.
The next step to compare each method and define the most suitable approach, relative error (RE) ratio, was used in this research as equation (6) where \( X \) is the actual/referred value of quantity, and \( X_0 \) is the measured value of quantity [19].

\[
RE = \frac{X_0 - X}{X} \times 100\%
\] (6)

The actual/referred value (\( X \)) is derived from the output of cell-based model calculation via Petrel Software. Otherwise, the three numerical method is defined as the measured value (\( X_0 \)).

3. Results and Discussions

The study area derives from a geophysical and geological analysis of the hydrocarbon reservoir with established assumed boundaries. To calculate the volume of hydrocarbons, it is imperative to consider the oil contact limit, representing the lower boundary of the reservoir. The oil contact lower boundary defined in this study called the Lowest Closest Contour (LCC), as shown in Figure 3. This contact is obtained from the farthest contour around the estimated reservoir. Other than LCC, the oil compartment is also limited by the structural setting, which are faults found in the study area. The faults themselves are shown by the red line on the map in Figure 3.

After defining the oil reservoir, cell-based volume calculations were carried out on Petrel 2021 using the gridding method. The research area's volume is derived from the spatial constraints delineated by the upper and lower layers of the oil reservoir, forming a reservoir configuration. This hydrocarbon zone is 115.69 meters thick and has a geometry of 12,112.24 meters by 12,299.99 meters. There are several little cells placed across the space, up to 38,220 pieces in total. These cells create a spatial structure that is thought to represent a hydrocarbon zone, as illustrated in Figure 2.

On the other hand, the computational technique based on a Python script is used to calculate volume with numerical integration. There are 139 input points used, and each one contains geometric data such as the depth, thickness, and local X and Y coordinates. The result for the thickness map is illustrated in Figure 4.
After finding out the trend of the oil reservoir, the volume of the reservoir can be calculated. On the Petrel software, the calculation between volume of reservoir geometry and the oil reserves volume OOIP are done simultaneously by the system. Otherwise, in the numerical method, the geometry of reservoir is calculated first using equation (1) for trapezoidal method, equation (2) for pyramidal method, and equation (3) for Simpson’s rule. The number of iterations repeated until consecutive iteration results converge within a small margin. After that followed up with the OOIP calculation using equation (5).

Actually, the calculation of OOIP value can be influenced by a variety of factors, including porosity, water saturation, fluid volume factor, and facies. In this study, the facies component was disregarded. It is expected to be entirely filled with oil in a predetermined hydrocarbon zone. All of these limitations are set up in order to compare numerical integral computations and cell-based calculations on an equal term. The Table 1 shows the findings of the oil reserve calculation.

Table 1. Original oil in place volume

<table>
<thead>
<tr>
<th>No</th>
<th>Calculation Method</th>
<th>OOIP (million m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cell based model</td>
<td>8.55</td>
</tr>
<tr>
<td>2</td>
<td>Trapezoidal</td>
<td>8.63</td>
</tr>
<tr>
<td>3</td>
<td>Pyramidal</td>
<td>8.58</td>
</tr>
<tr>
<td>4</td>
<td>Simpson’s 3/8 rule</td>
<td>8.57</td>
</tr>
</tbody>
</table>

The data in Table 1 presents estimates of OOIP derived from different methods of calculation. The cell-based model yields an estimate of 8.55 million cubic meters of OOIP. Comparatively, the trapezoidal method estimates slightly higher at 8.63 million cubic meters, while the pyramidal method and Simpson’s 3/8 rule estimate 8.58 million and 8.57 million cubic meters of OOIP, respectively. These figures highlight variations in estimates resulting from different calculation methods. From the numerical integration techniques, the trapezoidal method yields the highest estimate and the cell-based model yields the lowest. Such variations underscore the importance of carefully selecting and understanding the implications of the chosen methodology in estimating hydrocarbon reserves.

To facilitate comparison between the reference value and the results of the proposed methods, the data is presented in relative error form. In this context, cell-based calculation stands out as the predominant method, recognized for its established technique in approximating the volume of oil reserves within geo-engineering. The trapezoidal method, the pyramid method, and Simpson’s 3/8 rule serve as the three numerical integration methods for measuring values. For clarity, the outcomes are displayed in Table 2.

Table 2. Relative error ratio between cell-based model and numerical integration

<table>
<thead>
<tr>
<th>No</th>
<th>Calculation Method</th>
<th>Relative error ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Trapezoidal</td>
<td>0.93</td>
</tr>
<tr>
<td>2</td>
<td>Pyramidal</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>Simpson’s 3/8 rule</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 2 reveals the relative error percentages associated with three different numerical integration methods compared to a reference value, cell-based model. The trapezoidal method exhibits a relative error of 0.93%, followed by the pyramidal method with 0.35%, and Simpson’s 3/8 rule with the lowest relative error of 0.23%. These figures highlight the accuracy of each method in approximating the volume of oil reserves when compared to the reference value. Notably, Simpson’s 3/8 rule demonstrates the highest level of accuracy among the three methods, with the smallest deviation from the reference value. Such insights into the relative error percentages offer valuable information for assessing the reliability and precision of numerical integration methods in estimating hydrocarbon reserves.

Moreover, referring to Chen et al. [25], the pyramidal method should show a minimum error could be proven by this study as the relative error output of the pyramidal method is just slightly different with the least error result, Simpson’s Rule. Furthermore, based on equation (4), then Simpson’s rule performs better than the trapezoidal method. These requirements have been met by the computations in this study’s results, proving that Simpson’s Rule is a superior method to others. In addition, it is noteworthy that Simpson’s rule manifests a comparatively lower error value than the other two methodologies. This observation is underscored by the comparative
analysis, wherein Simpson’s rule exhibits a higher resemblance to the values derived from cell-based calculations than the other two numerical integration techniques.

4. Conclusions

The primary findings arising from the techniques and results outlined in the study are summarized as follows. Initially, the cell-based volume calculations, which served as reference methodology, yielded an estimate of oil reserves amounting to 8.55 million m$^3$. Subsequently, employing the numerical integration method for volume calculations results in varied estimates, which are 8.63 million m$^3$ using the trapezoidal method, 8.58 million m$^3$ employing the pyramid method, and 8.57 million m$^3$ with the Simpson’s Rule 3/8 method. The error ratio of these numerical integration methods, when compared to the cell-based approach, remains under 1%. Notably, Simpson’s rule demonstrates superior performance compared to the other methods. Ultimately, the study underscores the feasibility of numerically determining hydrocarbon reserves through scripts written in the Python programming language.

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