



RETHINKING TEACHERS' PROFESSIONAL DEVELOPMENT: LESSON FROM EVALUATION IN MASTER PROGRAM OF MATHEMATICS EDUCATION

Mohamad Rif'at¹, Nurfadilah Siregar¹

¹Mathematics Education, Program in Education of Mathematics and Science,
Tanjungpura University
Email: mohammad.rifat@fkip.untan.ac.id

DOI: <http://dx.doi.org/10.26418/jpmipa.v13i2.49370>

Abstract

This research is based on the results of the evaluation of the last ten years related to the development of studies in the Mathematics Education Master Program, Tanjungpura University, through a partnership model to develop and implement national and international comparative studies on mathematics teacher education. The aims are to describe the methodology used in the national level study of teacher education through the Teacher Professional Education Program or Pendidikan Profesi Guru (PPG) - Teacher Education and Development of Studies in Mathematics and share key findings related to mathematics preparation for future teachers. The research applied a quantitative approach with cross-sectional survey method with students of the master program of mathematics education class of 2019/2022 and 2020/2021. The results of the study show that future teacher performance: (1) has less opportunity to learn than high achievers in geometry, functions, calculus, and groups with mastery of linear algebra, number theory, analytic geometry, introduction to probability, and statistics; (2) has less opportunity for those who study school mathematics in data representation, calculus compared to linear algebra, analytic geometry, introduction to calculus, calculus, probability, and statistics. Thus, it can be concluded that attention and emphasis are needed on the type and depth of learning materials given to future teachers who continue their studies at the master's level. This pattern represents an extension to better future secondary school teachers.

Keywords: Evaluation, Mathematics Teachers, Teacher Development, Mathematics Education



Received : 19/09/2021

Revised : 02/07/2022

Accepted : 05/07/2022

INTRODUCTION

The study of the preparation of future mathematics teachers is very important because their skills are a need of society and the global economy (Santamaría-Cárdaba et al., 2021; Xenofontos et al., 2021). However, studies in several countries report that mathematics teachers often show misconceptions, and it is stated that attention to school is perceived as underprepared (Ball & Bass, 2003; Fennema & Franke, 1992; Post et al., 1991). Teachers may know that they are teaching facts and procedures but are weak in conceptual understanding and have difficulty clarifying mathematical ideas or solving problems that require more than routine calculations (Ball, 1991). There are claims that teachers' knowledge of mathematics or its lack of knowledge can help explain student performance at a national or international level (Darling-Hammond, 2000; Howe & Ma, 1999; Ingersoll, 1999; Kilpatrick et al., 2001).

Recent research has begun to advance what mathematical knowledge is considered necessary in learning, despite knowing less to teach mathematics in secondary schools than in primary schools (Baumert et al., 2010; Dreher et al., 2018). A recommendation from the mathematics community, The Mathematical Education of Teachers, Conference Board of the Mathematical Sciences (CBMS) is to emphasize prospective mathematics teachers to develop a deep understanding of mathematics knowledge in teaching (Reid & Reid, 2017). The National Academy of Sciences review stated that "Successful mathematics teachers require preparation that includes

mathematical knowledge, how students learn, and pedagogy aligned with professional society recommendations" (National Research Council, 2010). Another recommendation is that "Quantitative and qualitative data about mathematics study programs are needed in teacher preparation institutions, such as research to improve understanding, the most effective preparation approach for developing effective teachers" (p. 124).

The study results by Evans (1969) stated that international comparative studies on education have helped educators view their education system more objectively because the factors that have the potential and are related to educational attainment must be defined in a standard way. Even & Ball (2009) note that preparing and retaining a quality teaching staff who can teach mathematics effectively is a worldwide challenge and that all researchers can benefit from the associated results worldwide. For example, a cross-country study by Britton et al. (2003) and Howe & Ma (1999) identify differences in teacher preparation that may explain some of the differences in school mathematics performance. Although this study used a relatively small sample or was not randomly selected, the results are limited in generalizability. In line with those study, this study aims to describe the methodology used in the national-level study of teacher education through the Teacher Professional Education Program –Teacher Education and Study Development in Mathematics, as well as share essential or significant findings related to the preparation of mathematics for future development teachers.

METHODS

Population and sample

The target population of future teachers consists of LPTK administering mathematics education courses in West Kalimantan, which have a profile as institutions that offer to learn opportunities to teach mathematics, explicitly preparing individuals who are qualified to teach high school mathematics. The four target populations in this study are FKIP Tanjungpura University, IKIP PGRI, STKIP Melawi, and STKIP Singkawang.

Using the stratified sampling method, the four institutions' educational programs' effective sample size was 400 future teachers. The adequate sample size means that the design must be as efficient (i.e., precise) as a simple random sample. The number of future teachers required at the selected institution is mainly based on the number of programs implementing institutions (eight) and the selection method used. Thus, no national-level sampling is used, which is usually designed for each individual with the same final estimation.

Instrument

Questionnaire for future teachers, containing a set of questions about learning opportunities, beliefs about mathematics, teaching, and learning. In addition, a questionnaire contains items for assessing mathematical knowledge for teaching in secondary school education units. In this case, the knowledge is defined as content knowledge and pedagogy (mathematics) after this referred to as PIM and PIP. The two instruments were developed in collaboration with mathematics lecturers and teachers and

used a comparative research design. Researchers compiled and reviewed the items from various previous studies (Cohen & Hill, 2000; de Ayala, 2008; Floden, 2002; Schmidt et al., 2007).

Analysist of the data

Previous research by Schmidt et al. (2007) related to developing learning opportunities (LO) indicators. Connections from previous research and theory demonstrate the validity of correlating facts about LO scales' content, meaningfulness, and appropriateness. Expert and exploratory techniques, confirmatory factor analysis, and scaling judgment Rasch were used to select items. The LO items were grouped into seven scales: (1) institutional mathematics level, (2) school mathematics level, (3) mathematics education or pedagogy, (4) education or pedagogy, (5) classroom diversity and reflection on practice, (6) school experience and practicum, and (7) coherence of teacher education programs. The first two scales represent opportunities to learn mathematics, while the other five cover other aspects of teacher education. This study only reported learning opportunities at representative institutions and school mathematics levels.

Mathematics knowledge for teaching is assessed using items in four domains: (1) numbers (whole numbers, fractions, decimals, integers, patterns and relationships, ratios, proportions and percents, and number theory, (2) geometry (geometric shapes, geometric measurements, locations, and movements), (3) algebra (patterns, algebraic expressions, equations, inequalities, formulas, and functions), and (4) statistics. In addition, these

items include a question about the topic further in each domain (for example, number irrational, accurate, and complex, and the subject of the calculus, analysis, linear algebra, and abstract algebra in the domain of algebra). The test items are arranged according to blocks; one set consists of five blocks for basic mathematics and another three blocks for junior high school mathematics. To avoid placing too much burden on each participant, the blocks were rotated between tests so that each teacher answered two blocks of items. The number of future teachers who take the test is directly linked to many blocks (instruments), each LPTK. There are more future teachers of essential mathematics mastery than secondary schools.

Item Response Theory (TRI) scale was used to create a score scale report that allows estimating the knowledge performance for each participant in this study (de Ayala, 2008). The process of calibrating test items to determine if the data agree with the TRI model. Less matched items are reviewed (i.e., combined score categories on items with multiple scores) or omitted from calculating reported scores. The resulting set of items is calibrated again, using rubric scoring, so that each LPTK contributes equally to the calibration. The results were used to estimate the final calibration LPTK as participants test on the same scale TRI and then transformed so that the international averages for the sample calibration on each scale PIM (and knowledge of mathematics pedagogy) is 500 and a standard deviation of 100.

The fulcrum is the preferential score on each score scale that is used to develop a description of test-takers on

the knowable and workable scale. Two sets of test items were identified for each fulcrum to develop a description of skills and knowledge at a given fulcrum. For each fulcrum, the mathematics educators (teachers and lecturers) who participated in the study specifically addressed the objectives of the international studies, analyzing a set of items and developing a description of the capabilities of the participants approaching that fulcrum. The results of the fulcrum description give a specific meaning to the points on the reported score scale.

RESULTS AND DISCUSSION

The study results show that between institutions according to specialization in mathematics as an expectation to prepare teachers to play the same role according to their qualifications. The four institutions prepare general teachers to teach mathematics in primary and secondary schools, and there is no program to become specialist mathematics teachers. On the other hand, what about the master's program in mathematics education or the teacher development program? Are they prepared to become specialists in mathematics teachers?

Findings from future teachers who answered the initial surveys are presented for four program groups: (1) Primary Mathematics Education; (2) Secondary Education Mathematics; (3) Higher Education Mathematics; and (4) Specialist Program in Mathematics. Consistent with Perry, Howard, and Tracey (in Goos et al., 2021), the groups are to identify beliefs about mathematics, beliefs about mathematics learning, and beliefs about mathematics teaching and used

to identify majority responses favoured by at least half the respondents.

The results contain an estimate of the proportion of future teachers in the sample from each LPTK who achieve or exceed each fulcrum. Thus, for the entire cohort of future teachers to be in the sampled target population. Empirical facts obtained from performance levels (i.e., probability greater than 0,70 or less than 0,50 or between 0,50 and 0,70) based on the projected TRI model, able to perform or not within the specified probability.

Findings show a substantial decrease in the prevalence of out-of-field teaching of mathematics. Our analysis also revealed little change over time in the pattern of deployment of teachers of mathematics, with out-of-field teachers most commonly assigned to teach Ordinary Level mathematics to students in non-examination years preparation of Higher Level mathematics classes, this practice gives insufficient priority developing students' interest, procedural fluency and conceptual understanding of mathematics. Research on effective mathematics teaching highlights the complexities and importance of teachers'

mathematical and pedagogical content knowledge in order to teach the content. A positive outcome is evidence of graduates being deployed to teach Higher Level mathematics, demonstrating evidence of value placed by the programme.

This study examines the opportunities for mathematics teachers to study further and school-level mathematics and their performance on the test. Due to the differences in teacher education programs in LPTK, overall comparisons are not the aim of the international study. In contrast, national-level results by level and specialization in mathematics from teachers expected in national standards prepare teachers to perform similar roles according to their qualifications. Among those qualified to become primary mathematics teachers, most of them can become general mathematics teachers (as they are today) who depend on LPTK, indeed not higher in the mastery of PIM through basic education. This type of teacher is prepared to teach either the lower or upper classes. The qualification of future teachers for basic education is to become a mathematics specialist (Tatto et al., 2012).

Table 1. Opportunities for future elementary education teachers to learn mathematics by domain level

Program	Domain LPTK		Level Domain	
	Average (%)	SE	Average (%)	SE
Basic Mathematics	0,52	0,01	0,64	0,01
Junior High School	0,23	0,01	0,37	0,01
Mathematics/Equivalent				
Mathematics High	0,45	0,01	0,44	0,01
School/Equivalent				
Specialist	0,55	0,01	0,74	0,01

In contrast, most future teachers at the junior high school level/equivalent are prepared as mathematics specialists. Some are qualified to teach up to junior high school/equivalent level, while others are qualified to teach up to high school/equivalent level and even higher (shown in basic analytic or symbolic abstraction skills). Thus, the findings regarding future teachers who answered in the initial survey are presented in Table 1.

Table 1 shows the average proportion of study areas according to future teachers in the higher-level mathematics and school mathematics domains. Learning opportunities range across a wide range of programs. For example, the group with the lowest average is a general mathematics teacher, and the highest is a specialist program. Among specialist programs, differences were also found in advanced mathematical knowledge. Half of the programs had an average of 50% or more specialist domains, and half had less than half.

Table 1 also shows that future teachers cover an average of at least 60% of the school mathematics domain. Among advanced

mathematics domains, for example, more than 80% of future teachers study number theory, and more than 70% study probability. At least 60% of future teachers learn calculus. More than 70% of the future teachers in the LPTK sampled studied linear algebra.

In the junior high school/equivalent and senior high school/equivalent program groups (up to class X), the high proportion of students across LPTK studied numbers and measurements, while geometry varied widely. However, approaching 100% is in the interest of following an upgrading program to become a specialist through geometry. In contrast, about 50% of primary education teachers who teach lower and upper grades (up to grade IV) have the opportunity to learn geometry in the LPTK program. Opportunities to study functions, probability, calculus, and structure are generally low, except the need for and linkage to specialist mathematics courses in all LPTK. In general, teacher education needs to shift towards a higher level and become more specialist, an emphasis on study materials on functions, data, calculus, and structure becomes important.

Table 2. Opportunities for future secondary education teachers to learn mathematics by domain

Program	Domain of LPTK		Level Domain of School	
	Average (%)	SE	Average (%)	SE
Up to class X	0,42 – 0,84	0,01 – 0,03	0,60 – 0,94	0,01 – 0,03
Class XI to class XII	0,63 – 0,95	0,01 – 0,04	0,71 – 0,04	0,01 – 0,03

Table 2 shows the coverage in advanced mathematics and school mathematics domains for future high school teachers. Learning opportunities vary within and across

programs, where being prepared to teach up to grade XI or XII generally contains a higher proportion of domains than those prepared to teach grade X. In the ability group to teach

up to grade X, future teachers master the above. 70% of advanced math.

The lowest proportion of the group is around 40%. In the cohort for grades XI through XII, future teachers in all other programs cover at least 70% of the area of mathematical knowledge in that domain, while nationally cover 90% or more of the advanced mathematics domain. On the other hand, Table 2 also shows that future high school/equivalent teachers in both programs generally have substantial

opportunities to study school-level mathematics, with an average of 70% or more in terms of domains up to grade X.

PIM is described in two parts. The first is a qualitative description developed for the PIM fulcrum. Second, describe program groups and LPTKs in the fulcrum, as presented in Table 3. Such descriptions allow for comparisons of performance levels across the programs studied.

Table 3. Description of statistics for PIM by group

Program	SE	SD	Fulcrum 1		Fulcrum 2	
			%	SE	%	SE
Basic Math	3,9 – 9,9	62,8 – 91,1	11,9 – 90,5	0,3 – 5,7	0,9 – 57,3	0,5 – 5,4
Lower Grade						
Elementary	1,9 – 7,6	51,7 – 84,2	60,7 – 100	0,3 – 5,1	6,3 – 93,2	0,9 – 3,2
Mathematics						
Upper Grade						
Middle School	2,1 – 5,9	48,1 – 74,0	60,6 – 96,5	1,4 – 5,3	4,0 – 68,7	0,7 – 2,8
Mathematics						
Specialist	1,8 – 7,5	53,5 – 92,5	91,7 – 98,3	0,9 – 2,1	28,1 – 91,0	1,4 – 7,0

Future teachers for basic education mathematics who answer tests and obtain a Fulcrum of 1 are generally successful in performing basic calculations on integers, understanding the properties of integer operations, and giving reasons related to the concept of odd and even numbers. They can solve problems on fractions. Then, they can visualize and interpret two- and three-dimensional geometric shapes and solve problems related to the circumference. They can also understand the use of variables and the concept of equivalence and solve problems in the form of mathematical expressions and simple equations.

Although teachers at Fulfillment 1 can apply arithmetic to integers in simple problem-solving situations, they tend to overgeneralize and find it challenging to solve abstract and multi-step problems. They are limited in understanding the concept of Least Common Multiple (LCM) and the number line. Their knowledge of proportions and multiplicative reasoning is still weak. They have difficulty solving problems involving coordinates and problems concerning the relationship between geometric shapes. Future teachers can make simple but complex deductions to reason through multiple statements and relations between several mathematical

concepts at this fulcrum. For example, determining whether subtraction of integers is associative, understanding that there are an infinite number of decimal places between two given numbers, finding the area of a triangle on a grid of paper, and identifying the algebraic representation of a numerical relation between three consecutive even numbers.

Future primary education teachers who answer a test that then places them at Fulcrum 2 can complete the tasks at Fulfillment 1 successfully. In addition, this group was more successful than the teacher at Fulcrum 1 in using fractions to solve story problems and recognizing examples of rational and irrational numbers. They know how to find the LCM of two or more numbers in a familiar context and recognize that some integers' arguments are logically weak. They can determine the area and perimeter of geometric shapes and understand class inclusions between polygons (polygons). Future teachers at Fulfillment 2 are also familiar with linear expressions and functions.

However, even though future primary education teachers at Fulcrum 2 can solve proportions, there are still reasoning problems regarding factors, multiples, and percentages. They have not been able to solve the area of an obtuse triangle that requires coordinate geometry. They were not familiar with quadratic or exponential functions. They had limited success in applying algebra to geometric situations, for example, writing an accurate statement about a map (or image) of a coordinate point (a, b) over the x -axis, identifying a set of geometric statements that uniquely defines a square, and describing the properties of a function

defined using the ratio of area and circumference of a circle.

Overall, future teachers at Fulfillment 2 generally performed well on the "know" test items and common problems of numbers, geometry, and algebra, classification of "applications." However, they had; more difficulty answering problems requiring "complex reasoning in its application or non-routine situations." From the two test items regarding PIM obtained from the basic education level survey, it was found that the percentage of mastery according to the four parts of the instrument was various.

The future teacher's score at Fulcrum 1 of the initial survey for correctly answering the four parts of the question is at least 70% probability, but less than 50% chance of getting the fourth part correct (hierarchical level of difficulty). In this case, teachers at Fulcrum 1 tend to overgeneralize associative properties. On the other hand, a score at or above Fulcrum 2 has at least a 70% chance of giving correct answers for all parts of the test item. Algebra items were more difficult for teachers at Fulfillment 1 and 2. In this case, algebra items were more difficult for teachers to complete at both fulcrum, i.e., 12% of the national sample for full credit in PIM items and an additional 22% who had to get a share of the credit. Teachers at Fulcrum 2 get less than a 50% chance of correctly answering non-routine items regarding expressions in the variables.

Table 3 shows the fulcrum and descriptive statistics for PIM achievements in the basic education program group. Fulfillment Point 1 represents a lower level of performance and corresponds to a score of 431 on the baseline PIM scale. Fulfillment

Point 2 presents a higher level and a scaled score of 516. Across program groups and within each of the participating LPTK, scores on the PIM scale vary widely. The distributions within each of the four groups overlap equally. That is, even though the LPTK scores were lower, there were some teachers who outperformed some teachers from the higher scoring LPTK.

There are the effects of participation in the programme, we followed Desimone's (in Goos et al., 2021) recommendation to interpret teachers' self-reports of behavioural change in the context of the epistemological and pedagogical beliefs elicited via other survey items. Our claims are supported by previous studies, conducted by Goos et al. (2021) that involved different cohorts of participants and different data sources and analysis methods. Finally, we are conducting comparative case study research involving structured classroom observations, interviews, and surveys of three groups of mathematics teachers: (i) those who have been upskilled via the program; (ii) those who are still teaching mathematics out-of-field; and (iii) those who have always been fully qualified and hence in-field. Initial analysis of classroom observation data reveals that the upskilled teachers may be adopting pedagogical practices that are more like those of in-field teachers than those who are still teaching mathematics out-of-field, especially in relation to promoting higher order thinking, problem solving, and connectedness between mathematical concepts (Goos et al., 2021).

CONCLUSIONS AND RECOMMENDATIONS

From the results and discussion, conclusions can be drawn: First, what is impressive is the finding of variations in teacher education structures. This is a challenge for researchers at every study step in designing sampling, analysing, and reporting. Understanding teacher education variations allowed the alternative thinking research team to report a "league ranking table" of LPTK performance. To pursue this goal, researchers should be able to present an analysis of data and reports to demonstrate the strength of the teacher preparation systems in different LPTK for future teacher knowledge while remaining sensitive to local variations in program goals and objectives, such as future teachers by level and degree of specialization.

A second important finding is a variation in PIM even within programs. The difference in mean scores in PIM between the highest and lowest achieving LPTK in each program group is between one and two standard deviations. The difference of the two standard deviations is substantial, comparable to the transformation of the 50th and 96th percentile scores in the groups. Most future teachers scored at or above the higher PIM fulcrum in the LPTK, with the top scores in each program group. Differences between LPTK in program clusters tended to be greater among secondary school groups than among primary education groups. Future primary education teachers are doing very well in reaching Fulcrum 1, but only 50% are reaching Fulfillment 2, i.e., generalists or specialists.

Close to 70% of the teachers are not working well in the middle school group. That is, the preparation of teachers to teach is lower for class X to the maximum. However, the teacher did better for middle school through grade XII in achieving Fulfillment 1, even though the score was not high. In contrast to all LPTKs, more than 60% of future teachers achieved Fulfillment Point 2, and more than 55% failed to reach Fulfillment Point 2. Teacher performance is commensurate with the overall learning opportunities. Future primary education teachers appear to have lower opportunities to learn in terms of higher achievements than teachers in geometry, functions, calculus and validation, and abstraction, and advanced mathematics in linear algebra, number theory, analytical geometry, introduction to calculus (calculus 1), probability, and statistics. Future high school teachers appear to have fewer opportunities to study school mathematics in data presentation, calculus and validation, and abstraction, and advanced mathematics in linear algebra, analytical geometry, introduction to calculus, probability, and statistics.

Thus, recommendations for education providers to be able to prepare teachers in various educational units in various LPTK who demonstrate high levels of achievement in international tests require or require a wider scope of fields and domains. On the other hand, the relationship between PIP and PIM is complicated. Among basic generalist teachers was lower, content knowledge of mathematics was stronger in some LPTK, and opportunities for learning advanced mathematics and school mathematics for future teachers were

also higher in the primary education group. However, the teacher got the lowest PIP, but not the lowest score in the PIM.

In the junior high school/equivalent education unit group, the highest average PIM performance was achieved by future teachers, the highest average PIM was achieved by teachers from the two LPTK, but only intermediate opportunities for learning advanced mathematics and school mathematics in teacher preparation. In contrast, future generalist teachers at one LPTK showed the highest level of PIP but the lowest average PIM compared to other LPTK. PIP and PIM are both potential mediators according to the context, program policies, and background of future teachers. For example, the tendency of LPTK to accept future teachers with medium and low performance. Therefore, the educational program does not require many opportunities to study school mathematics and high-level mathematics (advanced mathematics) compared to national and international class LPTK. In this case, the educational program showed low PIP levels were associated with high school and high school mathematics. However, it may mean that the program designer assumes that the content of mathematical knowledge has been taught beforehand.

Mathematics teacher education is also influenced by the system in the LPTK, namely whether it is strong or weak. Meanwhile, the level of administrative oversight is centralized or decentralized, whether the program is handled responsibly for various performances, and whether the philosophy of an LPTK views diversity

in mathematical knowledge as the value of diversity, both in the classroom and between preparations to become teachers. Thus, to understand the determinants of PIM, a more satisfactory model should be carried out.

Another contribution of this study is the database of national and international studies. The database and its documentation provide a shared language and, with fulcrum descriptions, share *benchmarks* for examining evidence-based teacher preparation programs in multiple contexts. Thus, it is available to other researchers or a second analysis to develop and test their various hypotheses. Thus, an important message that can be conveyed to teacher education and policymakers is that attention needs to be paid and emphasized regarding the type and depth of learning opportunities provided to future teachers. For example, future teachers from high-achieving LPTK (or even countries) generally provide opportunities to study higher-order mathematics, particularly geometry, continuity, and functions, school-level mathematics, specifically calculus, probability and statistics, and structure. This pattern represents an extension to better future secondary school teachers.

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