JURNAL PENDIDIKAN MATEMATIKA DAN IPA

Vol. 13, No. 1 (2022) h. 25-39

http://jurnal.untan.ac.id/index.php/PMP

ANALYSIS OF ERROR IN DETERMINING THE DISTANCE BETWEEN TWO POINTS IN THE CARTESIAN PLANE IN THE FIRST SEMESTER STUDENTS

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DOI: http://dx.doi.org/10.26418/jpmipa.v13i1.43259

Abstract

The purpose of this study is to describe the error in determining the distance between two points. This type of research is descriptive qualitative research. The subjects of this study were 6 first-semester students of the mathematics education study program at one of the universities in Malang City who experienced errors in determining the distance between two points. The research instrument consisted of the problem of the distance between two points and the interview guide. The data collected in this study were in the form of student worksheets and interviews. Student work data were analyzed based on the types of errors that aim to determine the characteristics of student errors. The results of the interviews were analyzed by coding words or sentences to find out the causes of student errors. The results showed that there were three errors in determining the distance between two points, namely: misconception caused by misunderstanding and intuitive thinking; miscalculation caused by lack of thoroughness, and fact errors caused by lack of thoroughness and lack of attention to the overall information contained in the problem.

Keywords: *Fact Error, Miscalculation, Misconception, Trigonometry.*

INTRODUCTION

Trigonometry is generally defined as a branch of mathematics that deals with triangles (Downing, 2009) and specifically discuss the functions of sine, cosine, and tangent (Lial et al., 2016). Trigonometry is a fundamental topic taught in high schools and colleges (Nejad, 2016). Trigonometry is a prerequisite course for calculus, vector analysis, and differential equations (Mustangin & Setiawan, 2021; Setiawan, 2021a, 2021b). Trigonometry is also taught at the high

Received **: 19/11/2020**
Revised **: 21/12/2021**
Accepted **: 04/01/2022**

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school level. Therefore, as prospective teacher, students must master the concepts in trigonometry courses.

However various research results showed that students still have errors and difficulties in learning trigonometry. The results of the study show that learning trigonometric ideas is difficult for students and the causes can be from various interrelated aspects (Wongapiwatkul & Laosinchai, 2011). The results showed that the students experienced difficulties in solving trigonometric equations because they had not mastered the concepts and principles (Imelda, 2018). The results of other studies show that students still experience fact errors, skill errors, conceptual errors, and principle errors in solving trigonometry problems (Abidin, 2012; Setiawan, 2021a, 2021b). One of the fundamental obstacles is that trigonometry is much abstract and non-intuitive (Nabie, Akayuure, Ibrahim-Bariham & Sofo, 2018). More specifically, student errors in solving trigonometric problems include: not paying attention to quadrants, confusing trigonometric formulas, and not connecting trigonometry with other concepts (Jaelani, 2017). So in general it can be said that students still have difficulties in learning trigonometry material which results in errors in solving trigonometric problems.

The problem of students' difficulties in solving trigonometric problems has attracted the attention of several researchers. One thing that can be done by researchers is to analyze student errors in solving trigonometry problems. The results of the analysis of student errors in solving mathematical problems have many benefits. By knowing students' mistakes, the teacher can reduce these errors by improving mathematics learning in class (Setiawan, 2020c, 2020a, 2020b). In addition, discussions about student errors can reveal student misunderstandings (Ball, 1993), for better achievement (Kazemi, 1998), and can trigger student involvement in learning (Webb & Mastergeorge, 2003). So in general it can be said that research on the analysis of student errors in solving math problems is also important to do to improve student understanding.

The importance of conducting research on student and college student errors in solving mathematical problems (especially trigonometry problems) has caused several researchers to be interested in conducting research. For example, research that analyzes errors of first semester students of mathematics education study program in solving trigonometry problems, namely research (Abidin, 2012; Imelda, 2018; Jaelani, 2017; Setiawan, 2021a, 2021b). The results of the study indicate that the first semester students still experience conceptual errors, skill errors, fact errors, and principle errors. These errors are used in this study to classify student errors in determining the distance between two points.

However, the previous research did not discuss the error in the material of the distance between two points. The material on the distance between these two points is very important to be given to students. This is because the distance between two points is used in trigonometry as a basis for finding the radius of a circle, which is then used to understand circle trigonometry. However, the results of preliminary

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research show that some first semester students still experience errors in determining the distance between two points. The errors in determining the distance between these two points are important for further analysis. By knowing these errors, students can correct their misunderstandings in determining the distance between two points and can learn the distance between two points at the high school level correctly.

Different from previous research (Abidin, 2012; Imelda, 2018; Jaelani, 2017; Setiawan, 2021a, 2021b), where this study aims to describe the errors of first semester students in determining the distance between two points on the Cartesian plane and the causal factors. The results of this study will be useful for correcting the errors of students and college students in determining the distance between two points on the Cartesian plane.

METHODS

This type of research is descriptive qualitative research with a case study approach to 6 first semester students who experience errors in determining the distance between two points. The subject selection process was carried out by asking 81 first semester students of the mathematics education study program at one of the universities in Malang City to solve the problem of distance between two points (see Figure 1). From 81 students, 19 students experienced errors in determining the distance between two points. Furthermore, these 19 students will be grouped based on work errors (see Table 1). From the results of grouping errors obtained 6 types of errors made by students in determining the distance between two points. Of the 6 types of errors will be selected research subjects from each type of error, in order to obtain 6 research subjects.

The data collected in this study consisted of the results of student work and transcripts of interview results. The procedure for collecting student work is carried out in accordance with the procedure for selecting the subjects of this study. The procedure for collecting data from interview transcripts was carried out in two steps. The first step is to conduct interviews with research subjects and record interview activities. The second step is to transcribe the interview tape verbatim.

The research instrument consisted of two instruments, namely a question of the distance between two points on the Cartesian plane (see Figure 1) and an interview guide. Both of these research instruments were developed by the researchers themselves.

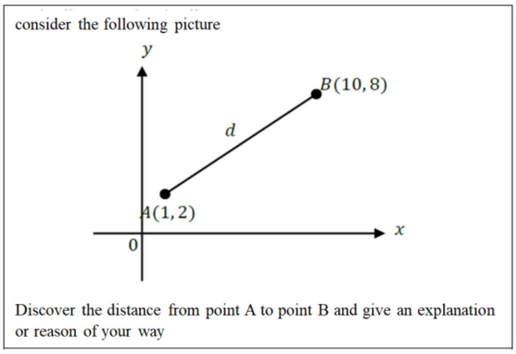

Figure 1. Research instruments

From Figure 1 it can be seen that the instrument in the form of question of distance between two points can be used to identify student errors in determining the distance between two points. For example, if students only see straight lines (without further

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analysis), they will get stuck to the equations of straight lines or gradients. In addition, if students only pay attention to point B, they will be trapped in the distance between point O(0,0) and point B.

In addition, the results of expert validation test by two lecturers of Mathematics Education at the Islamic University of Malang using expert judgment techniques showed that the instrument was valid to know student errors in determining the distance between two points (Setiawan, 2020c). The calculation error in this study is defined as error in performing arithmetic operations. While the fact error in this study is defined as error in understanding symbols in mathematics and also error in understanding the problem (Oktaviani, 2017; Muthukrishnan et al., 2019; Setiawan, 2020c). The selection of this error is based on the characteristics of instrument used in this study. The instrument used involves understanding the concept of distance between two points, calculations to produce distance between two points, and facts about symbols or information contained in the problem. The student error classification framework in this study can be seen in Table 1.

Table 1. Error classification framework

Error Type	Error Indicators
Concept Error	Mistakes the concept of distance between two points
	Mistakes in determining interrelated concepts
Count Error	Error determining the result of arithmetic operation
Fact Error	Error identifying information in the question

Analysis of interview transcript data was carried out by coding words or sentences that indicated the factors causing errors made by students. Through this interview, students' misconceptions in determining the distance between two points can be identified (Kalyuga, 2009). By analyzing the results of this work and interview transcripts, student errors in determining the distance between two points and the causal factors can be identified.

RESULTS AND DISCUSSION

The results of this study indicate that of 81 students of the first semester of mathematics education who completed the two-point distance problem, 19 students answered incorrectly. This means that 23% of the 81 students experienced errors. Of 19, 5 students did not answer. After being confirmed from the five students, 1 student said he forgot how to solve it, 1 student said he was still confused about solving the problem, 1 student said it was difficult to determine the distance because he still didn't understand how to determine the distance between two points, 1 student said that he didn't understand the solution steps, and 1 other student said that the time provided was not enough, so he did not answer the question about the distance between two points. Furthermore, from the 14 students who answered incorrectly, the student error classification was obtained in determining the distance between two points which can be seen in Table 2.

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Table 2. Results of error classification in determining the distance between two points

Error Type	Error Indicators	Error in determining the distance between two points	No of students
Concept Error	Misunderstanding the concept of distance between two points	Error understanding the coordinates	2
		Error understanding the distance as the coordinates	2
	Error in determining related concepts	Error using gradient to determine distance between two points	4
Count Error	Error in determining the result of arithmetic operation	Errors in determining the results of subtraction, addition, and the result of the square root	3
Fact Error	Error in identifying questions	Error re-writing the question	2
		Error not paying attention to starting point	1

After classifying the errors, the next step is to analyze the results of the work and interview transcripts by taking one student as the subject of each error made by the student in solving the problem of the distance between two points on the Cartesian plane. The purpose of this interview is to find out the cause of the subject's error. Each error in Table 2 is described below.

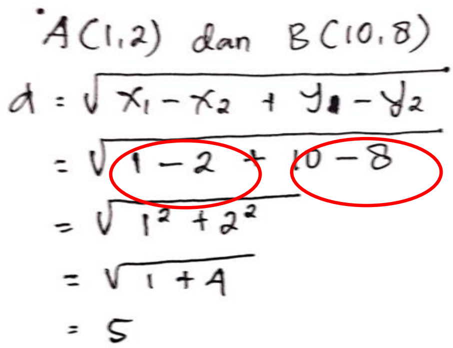

Figure 2. Error understanding point coordinates

Concept Error

The first misconception is an error in understanding the coordinates of a point. Students understand that the coordinates of point A (x_1, x_2) and coordinates of point B (y_1, y_2), then the student has an error in substituting the value of $(x_1 - x_2)$ and $(y_1 - y_2)$ which results in an error in determining the distance between two points. Of the two students who made this mistake, one student was selected as the first subject (S1) of this study. The results of the work of the first subject can be seen in Figure 2.

Figure 2 shows that the formula for the distance between two points used by the subject is still wrong, namely $d = \sqrt{x_1 - x_2 + y_1 - y_2}$. The mistake was not writing the brackets and the square of the respective subtraction operation. In addition, the subject is also wrong in substituting the value of $(x_1 - x_2)$ and $(y_1 - y_2)$, where the subject error is substituting point A (1, 2) to value $(x_1 - x_2)$ and substituting the value from point B (10, 8) to value $(y_1 - y_2)$. Factors causing subject error can be identified from the following interview excerpts.

P : *Where did you get $x_1 - x_2 = 1 - 2$ and $y_1 - y_2 = 10 - 8$?*

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S1 : *I* ***got it from point A which is 1 and 2, and point B which is 10 and 8.*** *From point A (1,2) input to $x_1 - x_2$ so 1 minus 2. While $y_1 - y_2$ in point B (10, 8), so 10 minus 8. It is as far as our understanding.*

From the interview transcript, it can be seen that the factor causing the subject to make a mistake in understanding the coordinates of a point is a misunderstanding by thinking that point A are x_1 and x_2 and point B are y_1 and y_2. So in general it can be concluded that the cause of errors in understanding point coordinates is a misunderstanding in understanding point coordinates.

The second misconception is the misunderstanding of distance as a coordinate point. As result of this misunderstanding, students write down the distance in the form of coordinate points which results error in determining the distance between two points. There were two students who made an error using the coordinates in determining the distance between two points. From the two students, it was confirmed that 1 student got an answer from his friend, and the other student worked on his own. Furthermore, this student who worked on his own was chosen as the second subject (S2) whose work results can be seen in Figure 3.

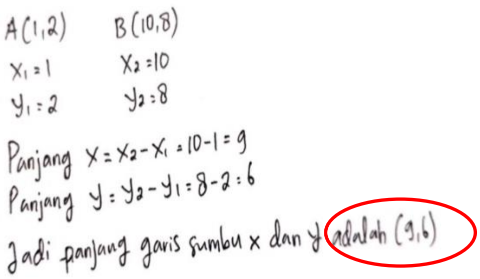

Figure 3. Error writing distance as coordinate

From Figure 3 it can be seen that the subject wrote down the distance between two points, namely the distance from point A to point B with coordinate points, namely (9, 6). Furthermore, to find out the cause of this error, the researcher conducted an interview with the second subject. The following is an excerpt of the interview transcript with the second subject.

P : *Please explain how you solve this problem?*

S2 : *I assume A(1, 2) therefore $x_1 = 1$ dan $y_1 = 2$ and for point B(10, 8) i assume $x_2 = 10$ and $y_2 = 8$. So to find out the length by $x_2 - x_1 = 10 - 1 = 9$ and $y_2 - y_1 = 8 - 2 = 6$. Therefore* ***the length from point A to point B is (9, 6) where $\mathbf{9 = x}$ and $\mathbf{6 = y}$.*** *It is as far as I understand*

From the snippet of the interview, it can be seen that the cause of subject making an error by writing the distance as a coordinate point is due to the subject's misunderstanding. The subject understands that the distance from point A to point B is the distance x and distance y, so the subject writes in the form of point coordinates. So in general the cause of the misunderstanding of distance as the coordinates of a point is the misunderstanding of the distance between two points as distance x and distance y.

The third misconception in determining the distance between two points is to use a gradient to determine the distance between two points. As a result of using this gradient, students experience errors in determining the distance between two points. There are 4 students who experience this error. Of the four students, it was confirmed that 2 students said they forgot the

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formula for the distance between two points, so they used a gradient to determine the distance between two points. One student said that he saw a straight line, so he thought of defining a gradient. One other student was chosen as the third subject (S3) in this study. The results of the work of the third subject can be seen in Figure 4.

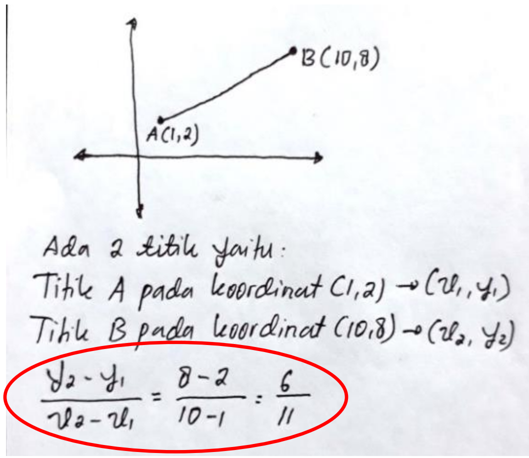

Figure 4. Using gradients to determine the distance between two points

From Figure 4 it can be seen that the subject uses a gradient to determine the distance between two points in the Cartesian plane. The cause of the subject's error in using gradients can be seen from the following interview transcript.

P : *Please explain the method you used to solve this problem?*

S3 : *Coordinates of point A (1, 2) for example x_1 and y_1 . While in point B(10, 8) for example x_2 and y_2 . Because on that number line* ***equation of straight and linear line*** *so that i* ***connected to the formula*** $\frac{y_2-y_1}{x_2-x_1}$*, so that the value of y_2 is 8 minus the value of y_1 is 2 per x_2 is 10 minus the value of x_1 is 1, the result is $\frac{6}{11}$. So that the distance between point A to point B is $\frac{6}{11}$.*

From the interview excerpt above, it can be seen that the cause of subject's error in using the gradient formula to determine the distance between two points is seeing the distance between two points as a straight line which is then intuitively connected with a gradient. The subject does not analyze further what is needed to find the distance between two points. So in general it can be said that the cause of subject using gradient in determining the distance between two points is because the subject uses intuitive thinking, which is directly seeing the line that is connected to the equation of straight line which is then connected to the gradient.

Count Error

There are 3 students who make arithmetic errors or arithmetic operations errors in determining the distance between two points. One student wrong in doing subtraction value $y_2 - y_1$, One student experienced an error in the sum of the results of square, and one student made an error in determining the result of square root. Students who experience errors in determining the result of this root are used as the fourth subject (S4) whose work results can be seen in Figure 5.

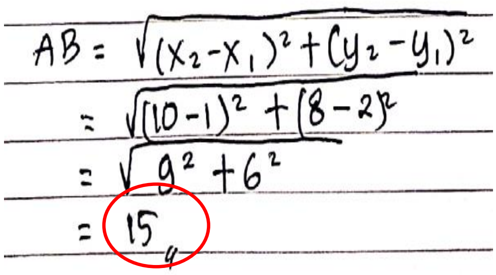

Figure 5. Error calculating root result

From Figure 5 it can be seen that in calculating the value from $\sqrt{9^2+6^2}$, the subject experienced an error, namely 15. The following interview transcript shows the factors causing this error.

P	:	*Why the result from* $\sqrt{9^2+6^2} = 15$*?*
S4	:	*Yes sir, it's wrong operation. Actually it's square root of 9 to the power of 2 plus 6 to the power of 2 equals the square root of 81 plus 36 equals the square root of 117. But yesterday I was in a hurry, so I* ***immediately removed the root and the square****, so nine plus six equals 15.*

From the interview footage, it can be seen that the factor that caused the subject to make errors in calculating was because the subject worked quickly, so the subject immediately removed the roots and squares. Other students who also experienced errors in calculations also said that they were careless because they were in a hurry to solve problems. So in general the cause of calculation errors are careless and in a hurry in solving problems.

Fact Error

The first fact error is an error in writing the question. As a result of being wrong in writing questions, students experience errors in determining the distance between two points. There were 2 students who made a mistake in writing the questions, namely wrong in writing one of the known coordinate points. One of the students was chosen as the fifth subject (S5) in this study. The results of the subject's work whom made mistakes in writing questions can be seen in Figure 6.

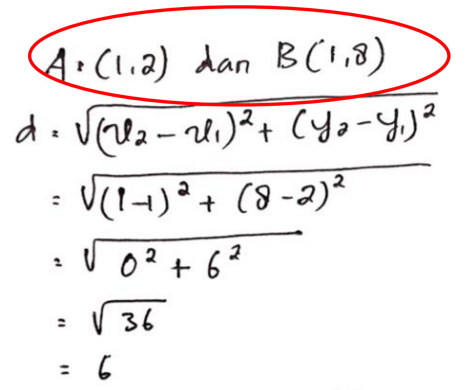

Figure 6. Error writing question

From Figure 6 it can be seen that the subject was wrong in writing the coordinates of point B with B (1, 8) which should be B (10, 8). As a result of this known error in writing, the result of the subject's work is wrong. The factors causing the error in writing this question can be seen from the following interview transcript:.

P	:	*Where did you acquire* $B(1,8)$*?*
S5	:	*From the question, sir, (then the subject looks at the question again) ... Sorry sir, I wrote the question wrong, so my answer was wrong, sir. Because* ***I did it in a hurry, sir and was careless***

From the interview transcript, it shows that the cause of subject writing the question incorrectly is that the subject is in a hurry to solve the problem and is careless. As a result of this carelessness, the subject experienced an error in determining the distance between two points. So in general it is found that the cause of subject making mistakes in writing the questions is because the subject is not careful in writing what is known in the problem.

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The second fallacy of fact is not paying attention to the starting point of the distance between two points. There was only one student who experienced this error, which was then used as the sixth subject (S6) in this study. The results of the work of the sixth subject can be seen in Figure 7.

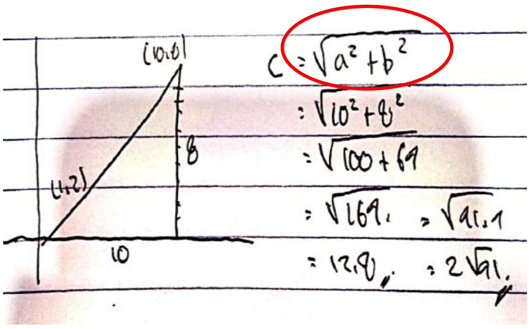

Figure 7. Error not paying attention to starting point

From Figure 7 it can be seen that the subject determines the distance from point A (1, 2) to point B (10, 8) using only point B (10, 8), while point A is considered as (0, 0). The following snippet of the interview transcript shows the factors causing the error.

P : *Please explain how you solve this problem?*

S6 : *I used Pythagoras, which is known to be asked to find the distance. So I think it's like a angled triangle. For* ***the a, it's 10 and the b is 8. So I immediately used the Pythagorean formula.*** *I think that's the way sir*

From the interview footage, it can be seen that the reason the subject made a mistake by not paying attention to the starting point was because the subject directly made angled triangle with the length of the upright sides being point B (10, 8), so the subject experienced an error in determining the distance between two points. So in general the cause of subject's error that does not pay attention to the starting point is the subject's lack of attention to the information contained in the problem, where the subject directly makes angled triangle with the length of the upright sides being the coordinates of point B.

Based on the results of study obtained description of the error in determining the distance between two points along with the causative factors can be seen in Table 3.

Table 3. Description of error determining distance between two points and causes

No.	**Error Type**	**Description**	**Cause Factor**
1	Concept Error	Conceptual errors in determining the distance between two points include: (1) an error in understanding the coordinates of a point, (2) an error in understanding the distance as a coordinate point, and (3) using a gradient to determine the distance between two points.	The causes of respective misconceptions include: (1) misunderstanding in understanding the coordinates of point A as (x_1, x_2) and point B as (y_1, y_2), (2) misunderstanding in understanding distance as distance x and distance y, and (3) Intuitively directly connect straight lines with equations of lines and gradients.

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2	Count Error	A calculation error in determining the distance between two points is an error when performing arithmetic operations in determining the distance between two points.	The cause of this calculation error is less thorough and in a hurry to solve the problem.
3	Fact Error	Fact errors in determining the distance between two points are errors in writing questions and errors not paying attention to the starting point of the distance between two points.	The cause of this fact error is not being careful in writing the questions again and paying less attention to the information contained in the questions.

The contribution of the results of this study is to develop a theory of errors in solving mathematical problems in the matter of distance between two points on the Cartesian plane. From the results of work analysis of students who experienced errors when determining the distance between two points, three types of errors were obtained, namely conceptual errors, arithmetic errors, and fact errors. Procedural errors were not found in determining the distance between two points, this is because the method of determining the distance between two points emphasizes conceptual knowledge rather than procedural knowledge. In addition, the principle error in the form of an error in applying theorems, arguments, properties or formulas in solving the problem of distance between two points was also not found. This is because to determine the distance between two points using only one theorem, namely the Pythagorean theorem. The results of this study are in accordance with the results of previous studies which showed that students in solving trigonometry problems still experienced fact errors, skill errors, conceptual errors, and principle errors (Abidin, 2012; Imelda, 2018; Jaelani, 2017; Nabie et al., 2018). However, the results of this study extend the results of previous studies by explaining the errors in the material of the distance between two points and their causal factors.

The first error in determining the distance between two points is a conceptual error. The first misconception that most students make in this study is using a gradient to determine the distance between two points. The use of gradients in determining the distance between two points is due to intuitive thinking, where the subject views the distance between two points as a straight line and directly relates it to the gradient. The second misconception is understanding distance as a coordinate point caused by a misunderstanding of distance, where the subject understands the distance between two points as distance x and distance y. The third misconception is the misunderstood point of coordinates. Misunderstanding the coordinates of this point is caused by a misunderstanding that understands point A as point (x_1, x_2) and point B as point(y_1, y_2). So in general it can be

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said that the cause of the misconception in determining the distance between two points is misunderstanding and intuitive thinking without doing analysis. The results of this study are in accordance with the results of previous studies which show that misunderstandings and prejudices against information can cause students to make conceptual errors in solving problems (Setiawan, 2020c). The results of previous research also show that this trigonometry course is considered abstract by students and requires a lot of analysis (Nabie et al., 2018). The results of this study are also in accordance with the results of previous studies which showed that students who had low abilities used irrelevant knowledge in solving problems (Setiawan, 2020a). In addition, thinking that is based only intuitively without doing analysis can cause students and college students difficulties or even errors in solving problems (Setiawan, 2020b, 2020d, 2020f; Setiawan et al., 2020). The results of this study extend the results of previous studies by showing that misunderstandings and only intuitive thinking (without doing analysis) can cause conceptual errors in determining the distance between two points. This is also supported by the opinion of experts who say that the information given to someone cannot be understood because of misunderstanding and prejudice (Sessa et al. in Kalyuga, 2009).

The second error in determining the distance between two points is a miscalculation or arithmetic operation error. The cause of this calculation operation error is due to lack of accuracy or in a hurry to solve the problem. The third error in determining the distance between two points is a fact error consisting of an error in writing the question and an error not paying attention to the starting point of the distance between the two points caused by lack of thoroughness and lack of attention to the question. So in general it can be said that the causes of calculation errors and fact errors are lack of accuracy and lack of attention to the information contained in the questions. The results of this study are in accordance with the results of previous studies which showed that this accuracy is important for someone to have in order to be successful in solving mathematical problems (Byers, 2009; Hästö et al., 2019; Setiawan et al., 2020). However, the results of the study expand the results of previous research by showing that the result of not being careful and paying less attention to the information contained in the questions is the occurrence of arithmetic operations errors and fact errors, especially in paying attention to the information contained in the questions.

The results of this study have implications for learning to overcome conceptual errors, calculation errors, and fact errors in determining the distance between two points. First, according to the cause of the emergence of misconceptions, namely due to misunderstanding and intuitive thinking, it is important to emphasize the different concepts of gradient, equation of straight line, distance between two points, and point coordinates. This is because the results of this study indicate that if you do not clearly distinguish these four things, you will experience a conceptual error. Various researchers also recommend

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that it is very important in learning mathematics to emphasize understanding concepts (Setiawan, 2020e; Setiawan & Mustangin, 2020a, 2020b; Setiawan & Syaifuddin, 2020). Second, in accordance with the cause of the arithmetic operation error, which is due to lack of accuracy, it is important to emphasize the accuracy in performing arithmetic operations by re-checking the answers that have been obtained. Rechecking the answers obtained can be done by checking the completion procedure (Rittle-Johnson & Star, 2007) and the results obtained. Third, in accordance with the cause of the emergence of fact errors, namely being less thorough and paying less attention to the overall problem or question given. It is therefore important to emphasize thoroughness and thorough attention to what is known in a mathematical problem or question. The results of previous studies have shown that selective processing can lead to errors in generalizing a pattern (Rivera, 2015; Setiawan, 2020b). The results of this study indicate that thorough attention is important for students and students to be successful in solving math problems. So in general it can be said that to reduce conceptual errors, calculation errors, and fact errors, it is done by emphasizing the understanding of a concept, the difference, and the relationship between a concept and another concept, by doing thoroughness, and paying attention to all the.

CONCLUSION AND RECOMMENDATION

The results of this study have contributed to the theory of errors in determining the distance between two points. There are three errors in determining the distance between two points, namely conceptual error, calculation error, and fact error. Conceptual errors are caused by misunderstanding and intuitive thinking, calculation errors are caused by lack of accuracy, and fact errors are caused by not paying attention to the information in the problem.

The limitation of this study is that it only focuses on one material, namely the distance between two points. The researcher recommends conducting further research by analyzing errors in various materials in the trigonometry course. By knowing the errors of students or students in understanding various materials, the results of the analysis of student or student errors can be used to improve students' and college students' understanding in studying trigonometry material.

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