PERHITUNGAN EFISIENSI MESIN LENOIR KLASIK DAN KUANTUM

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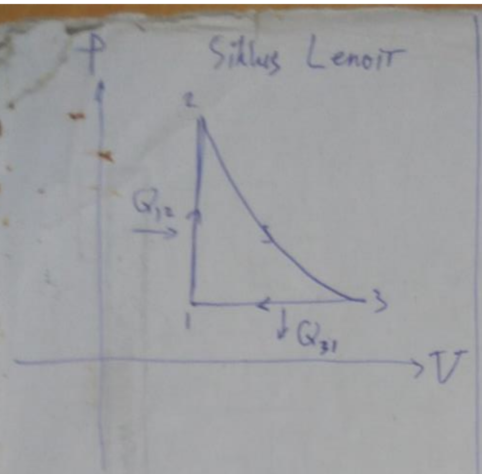


$1 \to 2$ (isokhorik):

$$Q_{12} = m c_v (T_2 - T_1)$$

$$W_{12} = \int_1^2 P \underbrace{dV}_{0} = 0$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} > 1$$

$2 \to 3$ (isentropik atau adiabatik reversibel):

$$Q_{23} = 0$$

$$\begin{aligned} W_{23} &= -\Delta U_{23} \\ &= -\int_2^3 dU \\ &= -m c_v \int_{T_2}^{T_3} dT \\ &= m c_v (T_2 - T_3) \end{aligned}$$

$3 \to 1$ (kompresi isobar):

$$Q_{31} = m c_p (T_1 - T_3) = -m c_p (T_3 - T_1)$$

$$\begin{aligned} \Delta U_{31} &= \int_3^1 dU \\ &= m c_v \int_{T_3}^{T_1} dT \\ &= m c_v (T_1 - T_3), \quad T_3 > T_1 \end{aligned}$$

$$\frac{P_3 V_3}{T_3} = \frac{P_1 V_1}{T_1} \Leftrightarrow \frac{T_1}{T_3} = \frac{V_1}{V_3} < 1$$

$$\begin{aligned} W_{31} &= Q_{31} - \Delta U_{31} \\ &= m c_p (T_1 - T_3) - m c_v (T_1 - T_3) \\ &= m (c_p - c_v)(T_1 - T_3), \quad T_3 > T_1 \end{aligned}$$

Cari W_{total}:

$$\begin{aligned} W_{total} &= W_{12} + W_{23} + W_{31} \\ &= 0 + m c_v (T_2 - T_3) + m (c_p - c_v)(T_1 - T_3) \\ &= m c_v T_2 - m c_v T_3 + m (c_p T_1 - c_p T_3 - c_v T_1 + c_v T_3) \\ &= m c_v T_2 - \cancel{m c_v T_3} + m c_p T_1 - m c_p T_3 \\ &\quad - m c_v T_1 + \cancel{m c_v T_3} \\ &= m c_v (T_2 - T_1) + m c_p (T_1 - T_3) \end{aligned}$$

atau

$$\begin{aligned} W_{total} &= |Q_{12}| - |Q_{31}| \\ &= m c_v (T_2 - T_1) - |-m c_p (T_3 - T_1)| \\ &= m c_v (T_2 - T_1) - m c_p (T_3 - T_1) \end{aligned}$$

Cari η:

$$\begin{aligned} \eta &= \frac{W_{total}}{Q_{12}} \\ &= \frac{m c_v (T_2 - T_1) - m c_p (T_3 - T_1)}{m c_v (T_2 - T_1)} \\ &= 1 - \frac{c_p (T_3 - T_1)}{c_v (T_2 - T_1)} \end{aligned}$$

atau

$$\begin{aligned} \eta &= \frac{W_{tot}}{Q_{in}} \\ &= \frac{Q_{in} - Q_{out}}{Q_{in}} \\ &= 1 - \left|\frac{Q_{out}}{Q_{in}}\right| \\ &= 1 - \frac{Q_{31}}{Q_{12}} \\ &= 1 - \frac{|m c_p (T_3 - T_1)|}{|m c_v (T_2 - T_1)|} \\ &= 1 - \frac{c_p (T_3 - T_1)}{c_v (T_2 - T_1)} \end{aligned}$$

dengan

$$\gamma = \frac{c_p}{c_v}, \quad r = \frac{P_2}{P_1}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Leftrightarrow T_2 = \frac{P_2}{P_1} T_1 = r T_1$$

$$\frac{T_2}{T_3} = \left(\frac{P_2}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = r^{1-\frac{1}{\gamma}}$$

$$\Leftrightarrow T_3 = T_2 r^{-(1-\frac{1}{\gamma})} = (r T_1) r^{-1+\frac{1}{\gamma}} = T_1 r^{\frac{1}{\gamma}}$$

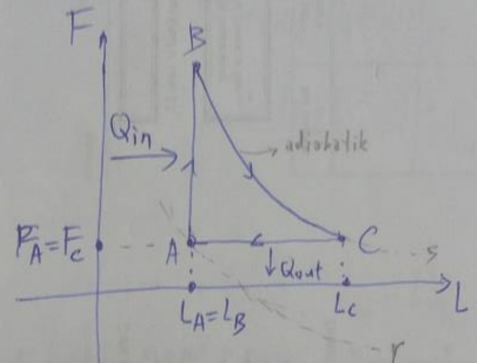
$$\eta = 1-\gamma\,\frac{T_1 r^{\frac{1}{\gamma}} - T_1}{T_1 r - T_1}$$
$$= 1-\gamma\,\frac{T_1 (r^{\frac{1}{\gamma}} - 1)}{T_1 (r-1)}$$
$$= 1-\gamma\,\frac{r^{1/\gamma}-1}{r-1}$$
$$= 1-\gamma\,\frac{\left(\frac{V_3}{V_1}\right)-1}{\left(\frac{V_3}{V_1}\right)^{\gamma}-1}$$

$P_2 V_2^{\gamma} = P_3 V_3^{\gamma}$

$P_2 = \left(\frac{V_3}{V_2}\right)^{\gamma} P_3 = \left(\frac{V_3}{V_1}\right)^{\gamma} P_1$

$r = P_2/P_1 = \left(\frac{V_3}{V_1}\right)^{\gamma}$

Siklus Carnoir Kuantum



$A \to B$ (isokhorik):

transisi keadaan $r \to s$

$$\psi_{AB}(x) = a_r \phi_r + a_s \phi_s$$

$$\Delta U_{AB} = E_B - E_A$$
$$= \frac{s^2\pi^2\hbar^2}{2mL_B^2} - \frac{r^2\pi^2\hbar^2}{2mL_A^2}$$
$$= \frac{s^2\pi^2\hbar^2}{2mL_A^2} - \frac{r^2\pi^2\hbar^2}{2mL_A^2}$$
$$= (s^2 - r^2)\,\frac{\pi^2\hbar^2}{2mL_A^2}$$

$$W_{AB} = 0$$

$B \to C$ (ekspansi adiabatik reversibel):

tidak mengubah probabilitas kehadiran partikel

$$\psi_{BC} = \phi_s$$

$$E_{BC} = \frac{s^2\pi^2\hbar^2}{2mL^2}$$

$$F_{BC} = \frac{s^2\pi^2\hbar^2}{mL^3}$$

$$W_{BC} = \int_{L_B}^{L_C} F_{BC}\, dL$$
$$= \frac{s^2\pi^2\hbar^2}{m}\int_{L_B}^{L_C}\frac{1}{L^3}\, dL$$
$$= \frac{s^2\pi^2\hbar^2}{m}\left[\frac{L^{-2}}{-2}\right]_{L_B}^{L_C}$$
$$= \frac{s^2\pi^2\hbar^2}{-2m}\left(\frac{1}{L_C^2} - \frac{1}{L_B^2}\right)$$
$$= \frac{s^2\pi^2\hbar^2}{2m}\left(\frac{1}{L_B^2} - \frac{1}{L_C^2}\right)$$
$$= \frac{s^2\pi^2\hbar^2}{2m}\left(\frac{1}{L_A^2} - \frac{1}{L_C^2}\right)$$

$C \to A$ (kompresi isobar):

transisi keadaan $s \to r$

$$\psi_{CA}(x) = a_r \phi_r + a_s \phi_s$$

$$|CA\rangle = a_r |r\rangle + a_s |s\rangle$$

Syarat normalisasi:

$$1 = \langle CA|CA\rangle$$
$$= (a_r^*\langle r| + a_s^*\langle s|)(a_r|r\rangle + a_s|s\rangle)$$
$$= a_r^* a_r \langle r|r\rangle + a_s^* a_s \langle s|s\rangle$$
$$= |a_r|^2 + |a_s|^2$$

$$|a_s|^2 = 1 - |a_r|^2$$

Ψφħ∆πΣ∮ds

maka gaya mekanik yang bekerja

$$F_{CA}(L) = \left[|a_r|^2 r^2 + |a_s|^2 s^2\right]\frac{\pi^2\hbar^2}{mL^3}$$

$$= \left[|a_r|^2 r^2 + (1-|a_r|^2) s^2\right]\frac{\pi^2\hbar^2}{mL^3}$$

$$= \left[s^2 + |a_r|^2 (r^2 - s^2)\right]\frac{\pi^2\hbar^2}{mL^3}$$

Pada proses isobar kuantum berlaku

$$F_C = F_{CA}$$

$$\frac{s^2\pi^2\hbar^2}{mL_C^3} = \left[s^2 + |a_r|^2 (r^2 - s^2)\right]\frac{\pi^2\hbar^2}{mL^3}$$

$$s^2\frac{L^3}{L_C^3} = \left[s^2 + |a_r|^2 (r^2 - s^2)\right]$$

$$L = \left[\frac{s^2 + |a_r|^2 (r^2 - s^2)}{s^2}\right]^{\frac{1}{3}} L_C$$

$|a_r| = 0 \to L = L_C$

$|a_r| = 1 \to L = \left(\frac{r}{s}\right)^{2/3} L_C = L_A,\ s > r$

$$F_{CA}(L) = \frac{s^2 \cancel{L^3}}{L_C^3}\,\frac{\pi^2\hbar^2}{m\cancel{L^3}} = \frac{s^2\pi^2\hbar^2}{mL_C^3} = \text{konstan}$$

$$= F_C = F_A$$

$$W_{CA} = \int_{L_C}^{L_A} F_{CA}\, dL$$

$$= \int_{L_C}^{L_A} \frac{s^2\pi^2\hbar^2}{mL_C^3}\, dL$$

$$= \frac{s^2\pi^2\hbar^2}{mL_C^3}\int_{L_C}^{L_A} dL$$

$$= \frac{s^2\pi^2\hbar^2}{mL_C^3}\left[L\right]_{L_C}^{L_A}$$

$$W_{CA} = \frac{s^2\pi^2\hbar^2}{mL_C^3}(L_A - L_C)$$

$$= \frac{s^2\pi^2\hbar^2}{mL_C^3}\left[\left(\frac{r}{s}\right)^{2/3} L_C - L_C\right]$$

$$= \frac{s^2\pi^2\hbar^2}{mL_C^3}\left[\left(\frac{r}{s}\right)^{2/3} - 1\right] L_C$$

$$= \frac{r^{2/3} s^{4/3}\pi^2\hbar^2}{mL_C^2} - \frac{s^2\pi^2\hbar^2}{mL_C^2}$$

Cari W_{tot}:

$$W_{tot} = W_{AB} + W_{BC} + W_{CA}$$

$$= 0 + \left(\frac{s^2\pi^2\hbar^2}{2mL_A^2} - \frac{s^2\pi^2\hbar^2}{2mL_C^2}\right) + \left(\frac{r^{2/3} s^{4/3}\pi^2\hbar^2}{mL_C^2} - \frac{s^2\pi^2\hbar^2}{mL_C^2}\cdot\frac{2}{2}\right)$$

$$= \frac{s^2\pi^2\hbar^2}{2mL_B^2} - \frac{3s^2\pi^2\hbar^2}{2mL_C^2} + \frac{r^{2/3} s^{4/3}\pi^2\hbar^2}{mL_C^2}$$

Cari Q_{in}:

$$Q_{in} = Q_{AB}$$

$$= \Delta U_{AB} + W_{AB}$$

$$= (s^2 - r^2)\frac{\pi^2\hbar^2}{2mL_A^2} + 0$$

$$= (s^2 - r^2)\frac{\pi^2\hbar^2}{2m}\cdot\frac{1}{\left[\left(\frac{r}{s}\right)^{2/3} L_C\right]^2}$$

$$= (s^2 - r^2)\left(\frac{s}{r}\right)^{4/3}\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$= \left(s^{10/3} r^{-4/3} - s^{4/3} r^{2/3}\right)\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$W_t = W_{AB} + W_{BC} + W_{CA}$$

$$= 0 + \left(\frac{s^2\pi^2\hbar^2}{2mL_A^2} - \frac{s^2\pi^2\hbar^2}{2mL_C^2}\right) + \frac{r^{2/3}s^{4/3}\pi^2\hbar^2}{mL_C^2} - \frac{s^2\pi^2\hbar^2}{mL_C^2}$$

$$= \frac{s^2\pi^2\hbar^2}{2m}\cdot\frac{1}{\left(\frac{r}{s}\right)^{4/3}L_C^2} - \frac{s^2\pi^2\hbar^2}{2mL_C^2} + \frac{r^{2/3}s^{4/3}\pi^2\hbar^2}{mL_C^2} - \frac{s^2\pi^2\hbar^2}{mL_C^2}\cdot\frac{2}{2}$$

$$= \frac{s^2\left(\frac{s}{r}\right)^{4/3}\pi^2\hbar^2}{2mL_C^2} - \frac{3s^2\pi^2\hbar^2}{2mL_C^2} + \frac{r^{2/3}s^{4/3}\pi^2\hbar^2}{mL_C^2}\cdot\frac{2}{2}$$

$$= \left[s^2\left(\frac{s}{r}\right)^{4/3} - 3s^2 + 2r^{2/3}s^{4/3}\right]\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$= \left[s^2\left(\frac{s}{r}\right)^{4/3} + 2r^2r^{-4/3}s^{4/3} - 3s^2\right]\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$= \left[s^2\left(\frac{s}{r}\right)^{4/3} + 2r^2\left(\frac{s}{r}\right)^{4/3} - 3s^2\right]\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$= \left[s^2\left(\frac{s}{r}\right)^{4/3} - r^2\left(\frac{s}{r}\right)^{4/3} + 3r^2\left(\frac{s}{r}\right)^{4/3} - 3s^2\right]\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$= \left\{(s^2-r^2)\left(\frac{s}{r}\right)^{4/3} + 3\left[r^2\left(\frac{s}{r}\right)^{4/3} - s^2\right]\right\}\frac{\pi^2\hbar^2}{2mL_C^2}$$

$$= \left\{(s^2-r^2)\left(\frac{s}{r}\right)^{4/3} - 3\left[s^2 - r^2\left(\frac{s}{r}\right)^{4/3}\right]\right\}\frac{\pi^2\hbar^2}{2mL_C^2}$$

maka

$$\eta = \frac{W_t}{Q_{in}}$$

$$= \frac{\left\{(s^2-r^2)\left(\frac{s}{r}\right)^{4/3} - 3\left[s^2 - r^2\left(\frac{s}{r}\right)^{4/3}\right]\right\}\cancel{\frac{\pi^2\hbar^2}{2mL_C^2}}}{(s^2-r^2)\left(\frac{s}{r}\right)^{4/3}\cancel{\frac{\pi^2\hbar^2}{2mL_C^2}}}$$

$$= 1-3\frac{\left[s^2 - r^2\left(\frac{s}{r}\right)^{4/3}\right]}{(s^2-r^2)\left(\frac{s}{r}\right)^{4/3}}$$

$$= 1-3\frac{s^2\left(\frac{s}{r}\right)^{-4/3} - r^2}{s^2-r^2}$$

$$\eta = 1-3\frac{r^2\left[\frac{s^2}{r^2}\left(\frac{s}{r}\right)^{-4/3} - 1\right]}{s^2-r^2}$$

$$= 1-3\frac{r^2\left[\left(\frac{s}{r}\right)^{2-4/3} - 1\right]}{s^2-r^2}$$

$$= 1-3\frac{r^2\left[\left(\frac{s}{r}\right)^{2/3} - 1\right]}{s^2-r^2}\cdot\frac{1/r^2}{1/r^2}$$

$$= 1-3\frac{\left(\frac{s}{r}\right)^{2/3} - 1}{\left(\frac{s}{r}\right)^2 - 1}$$

dengan

$$\left(\frac{r}{s}\right)^{2/3} = \frac{L_A}{L_C}$$

$$\frac{r}{s} = \left(\frac{L_A}{L_C}\right)^{3/2}$$

$$\frac{s}{r} = \left(\frac{L_C}{L_A}\right)^{3/2}$$

$$\eta = 1-3\frac{\left[\left(\frac{L_C}{L_A}\right)^{3/2}\right]^{2/3} - 1}{\left[\left(\frac{L_C}{L_A}\right)^{3/2}\right]^2 - 1}$$

$$= 1-3\frac{\left(\frac{L_C}{L_A}\right) - 1}{\left(\frac{L_C}{L_A}\right)^3 - 1}$$

$$= 1-\gamma\frac{r-1}{r^\gamma - 1}$$

dengan $\gamma = 3$

$$r = \frac{L_C}{L_A}$$